

Three-Dimensional Printing on a Budget: a Classroom-Friendly Technique for Viewing and Visualizing Solid Objects

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Abstract. Representing and understanding three-dimensional structures is a central problem in mathematics and science education. This paper describes a software system, *Spectre*, that can be used to print out a series of horizontal “slices” of three-dimensional objects onto transparency sheets. These transparencies may then be used in a (largely forgotten) half-century-old homemade device for displaying solid forms. We describe our software (and the accompanying physical device); discuss the advantages and drawbacks of our technique for three-dimensional representation; and outline directions for continuing and future work.

Introduction

Historically, one of the most difficult and pervasive problems in mathematics and science education involves creating understandable representations of three-dimensional structures. Finding an effective way to communicate (say) the arrangement of a complex molecule, or the shape of a galaxy, or the placement of various functional areas in the human brain, or the geometry of a complex set such as the Lorenz attractor, can tax the ability of even the best graphic designer or science illustrator. There is, moreover, a growing body of evidence that suggests that spatial visualization is a central skill in mathematical and scientific cognition [cf. Siemankowski and McKnight 1971, Miller 1984, Ferguson 1992, and Bryant and Squire 2001, among others]; so that, from the student’s point of view, understanding and exploring three-dimensional structure is a crucial part of the educational process.

There are a variety of plausible means of communicating or representing three-dimensional structures: static graphical representations (e.g., the sorts of illustrations seen in textbooks or popular science magazines), animated representations via computer graphics (e.g., interactive or manipulable renderings of three-dimensional structures), and physical models (e.g., molecular construction kits), to name a few. Each of these techniques has its own advantages and drawbacks. Scientific illustrations, while often ingenious and even beautiful, can still be difficult to interpret: consider, for instance, almost any diagram that attempts to show the location of the hippocampus in the brain, or the root structure underneath a growing plant. Computer animations are (arguably) an improvement, in that they can be viewed from multiple angles; but at any given moment, the object being viewed is still, unavoidably, a two-dimensional representation on a screen. Physical models have the advantage of tangibility, but they are generally domain-specific and often rather expensive.

This paper describes our progress in developing *Spectre*, a software system for use with an easily-constructed “homemade” device for viewing three-dimensional objects. Indeed, one can think of this device—in conjunction with the software—as a relatively inexpensive means of producing “true” three-dimensional illustrations. Like the techniques mentioned above, the method described here is not free of drawbacks (as we will describe shortly). Nonetheless, this technique likewise has some interesting advantages in comparison with traditional (and many experimental) techniques for visualizing three-dimensional objects. In the remainder of this paper, we describe both the device and our software-in-progress; we outline some of the advantages and disadvantages of our system; and we discuss both related work and directions for continuing and future development.

Spectre: a Description of the Device and Software

The basic idea behind our system involves the representation of three-dimensional objects through a series of planar “slices” through the object. A relatively simple homemade device for accomplishing this was described a half century ago in Cundy and Rollett’s invaluable book *Mathematical Models* [1951], and is

shown in Figure 1. The device is simply a vertical stack of slotted shelves onto which the user places a series of transparent sheets; each sheet depicts a horizontal planar slice of the desired three-dimensional form. In Cundy and Rollett's book—written well before the advent of the personal computer!—the authors suggest that the sheets be composed of glass (the type used for lantern-slides), and that the cross-sections be drawn by chinagraph pencil. Needless to say, this is a rather tedious and exacting chore for any teacher, student, or graphic designer who wishes to produce a three-dimensional drawing. Our software system, Spectre, is thus designed to facilitate the use of Cundy and Rollett's device by producing appropriate planar slices of three-dimensional objects which can then be printed on ordinary transparency sheets. By stacking the sheets within the device, one achieves a very reasonable (though somewhat “spectral”) view of the entire object.

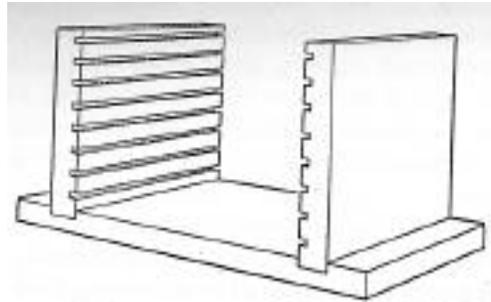


Figure 1. Cundy and Rollett's (1951) classroom device for viewing three-dimensional objects. The two vertical structures contain shelves on which transparency layers are placed.

Figure 2 depicts a typical project undertaken with Spectre. At the left of the figure, a screenshot of the software shows (in the smaller window) a three-dimensional rendering of several solid objects that the designer wishes to create. The larger window shows one particular “slice” through the three-dimensional scene; the user may, if she wishes, “page through” the series of slices on the screen to see how each one will appear. When the slices are finally printed out on transparency sheets and placed in the physical device itself, they produce the appearance of hovering solid objects, as shown toward the right of Figure 2.

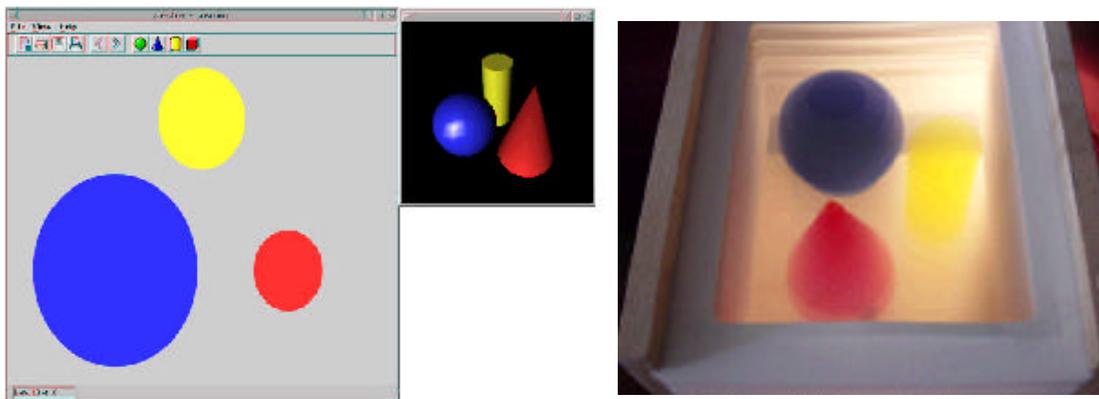


Figure 2. At left: a screen view of the Spectre system, showing (in the larger window at left) a “slice” through the set of solid objects rendered in the smaller window. At right: Spectre has produced a series of transparencies which may then be placed in the device sketched in Figure 1.

The current version of the Spectre software is still rather skeletal: the only fundamental types of solid objects that the user can currently place within a scene are cones, spheres, cylinders, and cubes. These objects may then be altered by the application of affine maps—e.g., for scaling about a given axis, rotation about an axis, and translation—so that (for example) a sphere might be transformed to an ellipsoid; and the

objects may be assigned any of a wide range of RGB colors. The overall result is admittedly far from a powerful, general-purpose system for three-dimensional construction. Nonetheless, within these constraints, it is possible to construct myriad “compound” objects made of multiple solid chunks: e.g., a “lollipop-like” figure could be composed of a sphere atop a narrow cylinder. Figure 3 shows still another example—a castle composed of multiple primitive Spectre objects. Our near-term goal is to develop the software to the point at which it can be made available free of charge over the World Wide Web.

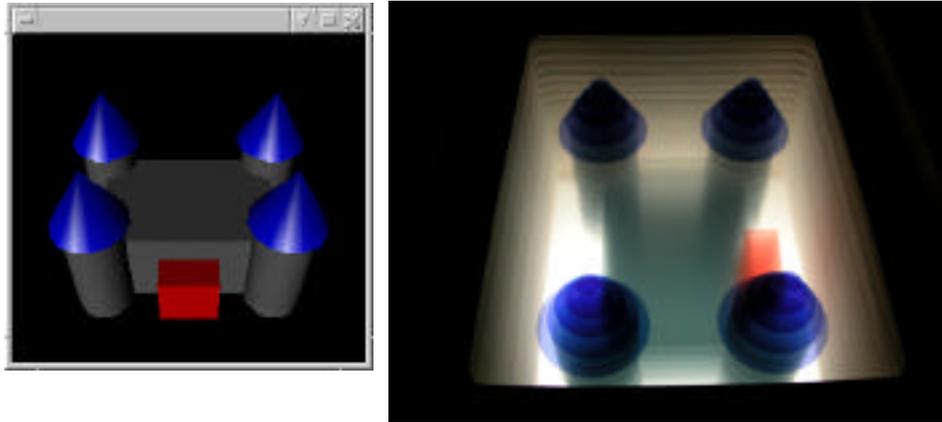


Figure 3. At left, Spectre’s screen rendering of a “castle”. At right, the castle in layered transparency form.

Advantages and Drawbacks of the “Spectre Technique” of Representation

Our experience to date with the Spectre system—even at its still early stage of development—suggests that the system has both interesting affordances and (in many cases, perhaps unavoidable) limitations. On the positive side, the objects rendered by Spectre are, in an important sense, “truly” three-dimensional: that is, a point that appears farther away on the rendered (transparency) object actually is farther away from the viewer. (Another way to put this is that the transparency objects really do have three distinct x, y, and z dimensions, though only the first two are rendered continuously.) As a consequence, these rendered objects permit types of interaction that even high-resolution screen renderings do not: one can walk entirely around a Spectre object, or gather a group of students around the device. Because the objects are rendered on transparency, their interior is visible (particularly when “softer” colors such as yellow are employed); this implies that solid objects with at least some interior detail may be adequately rendered by the device. Finally, although truly three-dimensional, Spectre objects do not share some of the (perhaps limiting) features of physical models: a Spectre object can be shown hovering in midair or “balancing” on a point or edge.

By the same token, there are definite limitations inherent in Spectre objects. First, and most obviously, because these rendered objects employ only a relatively small number of discrete slices, their resolution in the z-dimension is highly limited. Thus, objects with a great deal of “high-frequency” surface detail—e.g., a sea urchin—cannot be adequately rendered in this device. Conceivably, a higher resolution can be achieved by spacing the transparencies closer together, but this can only be achieved at the cost of both more transparencies per object and, more importantly, a lower degree of light transmission through the rendered scene; we are still seeking the proper tradeoff point between resolution and light transmission. A second limitation is that Spectre objects do not share important affordances of “true” physical objects: one cannot point to an arbitrary spot on the surface of an object with a physical pointer (or finger), because of the intervening sheets of transparency. There is also a “flip side” to the ability to see the interior of Spectre objects—namely, their *exterior* surface is less discernable. Thus, one would probably not (e.g.) represent a map of the globe on a Spectre object—even a higher-resolution object than our examples in Figure 2—since the visible interior of the object would interfere with the viewer’s ability to concentrate on the exterior surface.

Essentially, then, we see Spectre (in conjunction with Cundy and Rollett’s device) as one more technique to add to the teacher’s (or student’s, or designer’s) repertoire for three-dimensional representation. Many,

though assuredly not all, three-dimensional objects can be adequately represented this way. In the coming months, we plan to develop a “menagerie” of examples of three-dimensional illustrations rendered in Spectre.

Related, Continuing, and Future Work

Besides the techniques mentioned in the introductory paragraphs, there are still other (and in varying degrees, more experimental) methods for conveying information about three-dimensional objects. By now there are numerous designs for virtual reality systems capable of representing three-dimensional scenes at high resolution; but generally, these techniques require dedicated equipment for the user (e.g., glasses or gloves), and are far more expensive and complex than the method described here. One particularly interesting experimental “glasses-free” method for displaying three-dimensional objects, developed at New York University, is described in Alpert [2002]; but even this method (in contrast to Spectre), being screen-based, only permits certain viewing angles, and in addition it can only be used by one viewer at a time. A technically sophisticated (and relatively expensive) system for “true” volumetric display, based on projection onto a rapidly rotating screen, has recently been described [Favalora et al. 2002]; this is much higher-resolution than Spectre’s images, but far less accessible to teachers, students, and hobbyists.

In the near-term future, there are many extensions needed for Spectre before it can justifiably be called a widely useful system. Certainly, more “primitive” forms besides the current set (cones, cubes, spheres, and cylinders) are needed: additional basic forms such as the five regular solids, or general constructors for prisms and pyramidal forms, would be particularly useful. It would also be desirable to include interface techniques so that the user could (for example) produce representations of arbitrary functions from R^2 to R^3 (that is, from the x,y -plane to z); this could be especially useful for displaying (say) a potential function on a plane. More ambitiously, we hope to produce at least a few “special-purpose” three-dimensional scientific illustrations—possibly including a “cutaway” model of the earth showing the mantle and core, or a model of quantum orbitals—if only to explore the feasibility of representing a variety of unusual forms in our system.

There are still longer-term possibilities for experimentation with Spectre (or Spectre-like systems). One possibility might be to explore other forms into which flexible transparencies may be layered (e.g., nested cylindrical structures) to more easily represent specific types of three-dimensional forms. Transparencies might also be assembled into still more elaborate structures (such as “sliceforms” [Sharp 1999]) that could likewise be used to represent spatial structure. An especially ambitious prospect would be to explore methods by which Cundy and Rollett’s device could be used to display animated three-dimensional forms; in order to do this, it would be necessary to layer a series of transparent screens on which animated forms could be displayed in synchronized fashion (e.g., to display a rotating cube). Novel forms of “controllable” glass materials (such as the sort described in [Technology Review 2000]) could make this longer-term project feasible. In any event—even ignoring such futuristic notions—our experience with Spectre shows that computational media can lend new life to a half-century-old educational device.

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